Progression and Transfer of the Mathematical Modelling Perspective in Junior Secondary Mathematics: The Integrated Unit on Functions and Equations as a Case Study

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Abstract
The mathematical modelling perspective is one of the places where school mathematics meets the world beyond the classroom. Yet modelling tasks in many junior secondary classrooms are still compressed into a familiar routine: read the problem, write an expression, and calculate an answer. Students may handle familiar exercises without being able to state the assumptions behind a model, judge whether a result is credible, or decide how the model should change when a condition is altered. To address this fragmentation, the present study draws on a literature review, content analysis, and design research to describe modelling as five connected phases: identifying, constructing, interpreting, validating and revising, and transferring a model. Using the integrated unit on functions and equations, the paper reorganizes related content and proposes tasks that place different models in the same context, carry one model across contexts, vary constraints, infer relationships from data, and compare competing models. The aim is to help students attend to mathematical structure rather than remember only surface-level problem types. Assessment is organized around five dimensions and four developmental levels, with evidence drawn from worksheets, classroom explanations, modelling products, transfer tasks, and reflective notes. The outcome is a design framework rather than evidence of instructional effectiveness. Its practical value lies in making assumptions, domains of applicability, and transfer visible in unit goals, classroom tasks, homework, and assessment, while leaving the framework open to subsequent classroom testing and revision.

Keywords: *mathematical modelling perspective; integrated-unit instruction; learning progression; model transfer; functions and equations*

1. Problem Statement

1.1 Background

The Mathematics Curriculum Standards for Compulsory Education (2022 Edition) organize mathematical core competencies around three broad capacities: observing the real world with a mathematical eye, thinking about it mathematically, and expressing it in mathematical language. The mathematical modelling perspective is a concrete expression of these capacities. For junior secondary students, extracting quantitative relationships from a real situation is only a beginning. They also need to state the conditions on which a model depends, interpret results in context, and

judge whether the model should be revised or can be used in another problem. Modelling should therefore run through algebra, geometry, statistics, and integrated practice rather than remain an additional objective attached to a single chapter.[1] In practice, this developmental thread is not always visible. Textbooks are divided according to the logic of mathematical content, while classroom teaching commonly advances one lesson at a time. Students' modelling experiences can consequently remain tied to separate exercise types. They may learn to use a linear equation for a travel problem, a function to describe change, and a quadratic equation to maximize an area without ever comparing the variables, constraints, and representations involved. When the context changes, information is incomplete, or two models must be weighed against each other, a previously fluent procedure may offer little basis for judgment. Integrated-unit instruction offers one way to reconnect these experiences. It does not merely extend the number of lessons; it reorganizes content around a core idea and a set of consequential tasks. Functions and equations provide a particularly useful basis for such integration. An equation identifies an unknown state under specified conditions, whereas a function describes dependence and change between variables. The solution of an equation can also be interpreted as the x-coordinate of an intersection on a function graph. When students examine these connections within the same sequence of tasks, they are better placed to decide when an equation is useful, when a function is more informative, and how one representation can be translated into another.[10]

1.2 Research Questions and Methods

The paper addresses three questions. What developmental stages can describe an increasingly sophisticated mathematical modelling perspective at the junior secondary level? How can tasks in an integrated unit on functions and equations be arranged to support transfer? What evidence can teachers use to determine whether transfer has genuinely occurred? The study first reviews research on modelling competence and learning transfer, then uses content analysis to examine structural links between equations and functions. These analyses inform the proposed teaching framework, task examples, and assessment rubric. Because this is a theoretical and design-based study, its conclusions concern the coherence and practical usability of the proposal; instructional effects remain to be examined through subsequent classroom implementation.

2. Review of Related Research

2.1 Major Developments in Research on the Mathematical Modelling Perspective

International research on mathematical modelling has gradually moved from descriptions of a complete modelling cycle toward analyses of specific subcompetencies and process-related obstacles. Modelling has been treated both as a cycle from a real situation to a mathematical representation and back to a contextual interpretation, and as a set of abilities involved in simplifying a situation, establishing relationships, and validating results. From this perspective, students' difficulties often lie less in calculation than in the transitions among contextual information, a mathematical model, and the interpretation of a result. Recent reviews also caution that modelling competence is unlikely to develop through a few concentrated exercises; assessment needs to cover several phases and accumulate evidence over time. Research in China has often begun with a particular content area or instructional component. Wang Borui examined the complete modelling process through real-world problems involving quadratic equations in one variable. Yang Dongxue treated the curriculum domain of integrated practice as an important site for modelling and emphasized alignment among goals, activities, and assessment. Huang

Chunlan approached the issue through homework design and proposed principles concerning goal orientation, cognitive fit, and experience of the process. Although their immediate objects differ, these studies converge on one point: a modelling perspective cannot be developed simply through a teacher's explanation of a standard procedure. Students need to select information, express relationships, solve, and interpret for themselves before the method becomes part of their own understanding.[2][3][4] Project-based learning and interdisciplinary homework have broadened the forms that modelling tasks can take. Liu Chunjiang used a project structure to organize complex problems so that models functioned as tools throughout a sustained inquiry. Zheng Peijun designed tiered assignments around relationships among the sides and angles of right triangles, with attention to the connection among content, task demand, and everyday contexts. Yuan Meng's study of integrated-practice activities in statistics and probability, however, found that activity design, classroom organization, and multidimensional assessment remained prominent challenges for teachers.[6][7][9] The relationship between function teaching and the modelling perspective has also received considerable attention. Deng Zuye proposed a function-modelling pathway from identifying a problem and making assumptions to validation and application, emphasizing the role of functions in describing patterns of change. Huang Lihua showed how graphs, tables, manipulatives, and digital tools can make abstract relationships more accessible, while also arguing that visualization is educationally useful only when it supports analysis and interpretation. Bi Yueyue's analysis of high-stakes examination items further suggests that assessments confined to isolated calculations reveal little about students' ability to select and adjust a model in a complex situation.[5][8][11]

2.2 Limitations of Existing Research and the Focus of This Study

Several questions remain open. First, many studies are bounded by a single lesson, chapter, or activity and pay less attention to how a modelling perspective might develop across grades and content areas. Even when students complete an entire modelling cycle in one case, the experience may stay attached to that original task if the model does not reappear in varied contexts or become an object of explicit comparison.[14][16] Second, some designs devote most of their attention to constructing and solving a model, with less space for assumptions, validation, error, and revision. Any real-world model is conditional. If a lesson never asks about the range of a variable, the source of data, or the reasonableness of a simplification, students can easily mistake a mathematical expression for the reality it represents. Recognizing this distance is an important sign that a modelling perspective is becoming more mature. Third, evidence that transfer has occurred is often underspecified. Completing an application problem whose structure is almost unchanged shows that a student can reuse a procedure in a nearby case, but it does not yet show structural understanding. This paper therefore treats transfer as an advanced manifestation in the progression. Tasks place the same model in different contexts, compare different models within one context, and vary the conditions; students' reasons for choosing, processes of revision, and quality of interpretation are used to judge their independence.

3. Core Concepts and Theoretical Rationale

3.1 Mathematical Modelling Perspective, Learning Progression, and Model Transfer

In this paper, a model is a mathematical representation formed by selecting relevant features from a real object, quantitative relationship, or pattern of change. It may take the form of an expression, equation, function, graph, table, statistic, or probability structure. A mathematical

modelling perspective involves more than producing these representations. It includes an inclination to use them to understand a problem, explain their conditions and purposes, and validate or revise their results.[12][13][20] A learning progression describes a plausible pathway through which students' understanding of a core idea or capability may develop. It is not a relabelling of lesson steps; rather, it distinguishes typical performances by students at different levels on related tasks. The five levels proposed here follow a broad sequence but do not form a one-way line. Validation may send a student back to reconstruct a model, and transfer to a new context often requires variables and conditions to be identified again.[14][18] Model transfer occurs when students recognize a structural connection between an earlier and a new problem and use that connection to draw on, adapt, or rebuild a model. Changing only the numbers or names in a situation is not sufficient evidence of transfer. A stronger indication is that, even after the surface context or conditional structure changes, students can explain their choice of model and identify which parameters can be retained and which constraints require reconsideration.[16]

3.2 Theoretical Rationale

The design draws on four related theoretical perspectives. Constructivism treats knowledge as something developed through inquiry and social interaction, so classrooms need room for students to propose assumptions, reason with peers, and compare alternatives. Research on transfer warns against relying on surface cues and instead directs attention to shared structure and limits of applicability. Realistic mathematics education emphasizes the movement from a situation, through mathematization, toward a more formal representation. Integrated-unit instruction provides the curricular conditions for goals, content, tasks, and assessment to be organized around one core idea. The common implication is straightforward: students need to move repeatedly from reality to mathematics and back again by interpreting mathematical results. A lesson should ask why a model is appropriate before concentrating on how to carry out the associated calculations.[15][20]

4. A Five-Stage Learning Progression for the Mathematical Modelling Perspective

Drawing on curriculum requirements and the research reviewed above, this paper proposes a five-stage progression. The framework can be used both to decompose unit goals and to guide the reading of student work. Table 1 summarizes the central performance, observable evidence, and teacher support associated with each stage.

Table 1. Five-Stage Learning Progression for the Mathematical Modelling Perspective

Progression stage	Core performance	Typical evidence	Teacher support
Identify a model	Distinguishes variables, constants, conditions, and goals; detects a mathematizable relationship	Annotated problem statement, variable list, relationship diagram	Provides contrasting situations; asks which information is necessary or redundant

Progression stage	Core performance	Typical evidence	Teacher support
Construct a model	Selects a representation and builds an expression, equation, function, or graph/table	Definitions of variables, relational expressions, and conversions among representations	Encourages alternative definitions and multiple model forms
Interpret a model	Explains the contextual meaning of parameters, solutions, graphs, and calculated results	Units, contextual conclusion, interpretation of a graph	Requires every symbol and result to be returned to the situation
Validate and revise	Checks domains, assumptions, errors, and limits of applicability; revises the model	Reason for rejecting a root, residual comparison, revision proposal	Introduces anomalous data or changes a condition
Transfer a model	Recognizes structure in a new situation and invokes, adapts, or rebuilds a model	Cross-context solution, model comparison, transfer reflection	Designs same-model/different-context and same-context/different-model tasks

4.1 Identifying a Model: Finding Relationships in an Everyday Narrative

The first step in identifying a model is not to define an unknown, but to decide which quantities vary, which conditions impose constraints, and what may influence the target quantity. Students can first state the relationship in ordinary language and then draw a relationship diagram or construct a variable table. In a taxi-fare problem, for example, the base fare, base distance, and price per additional kilometre are parameters fixed by the tariff; distance travelled and total fare are variables; and the use of a piecewise tariff determines the basic form of the model.

4.2 Constructing a Model: Moving among Representations

Constructing a model means translating an identified contextual relationship into a mathematical representation. A single situation need not have only one representation. An equation may be convenient when a particular value is sought, whereas a function is more informative when the question concerns how cost changes with distance. Suppose the base fare is 8 yuan for up to 3 km and then increases by 2 yuan for each additional kilometre. Equation (1) gives the cost model, and Figure 1 shows its piecewise graph.

$$\begin{cases} C(x) = 8 & 0 < x \leqslant 3 \\ C(x) = 8 + 2(x-3) & x > 3 \end{cases} \tag{1}$$

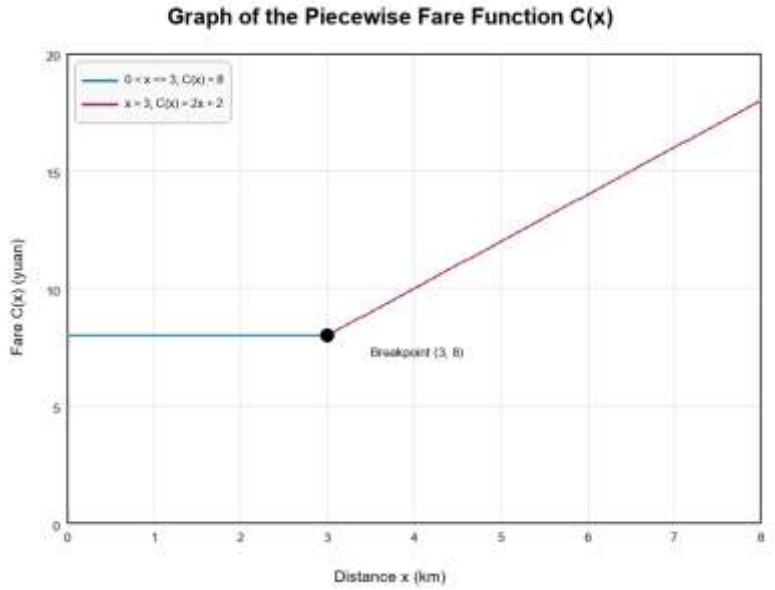


Figure 1. Piecewise Function for a Taxi Fare

If the question is extended to ask how far the taxi has travelled when the fare reaches 18 yuan, students can set the function equal to 18 and solve for the corresponding distance. Placing the two approaches side by side helps them see that the choice of model follows the question being investigated, not a fixed label attached to the exercise.

4.3 Interpreting a Model: Giving Meaning to Symbols and Results

To interpret a model, students must return its symbols and calculated results to the situation. The slope of a function graph can represent a rate of change, but the intercept does not always have a direct real-world meaning. A root of an equation is only a candidate that satisfies the mathematical relationship and must still be screened against contextual conditions. Classroom explanations can be organized around four questions—result, unit, object, and condition—so that an answer does not end with an isolated number.

4.4 Validating and Revising a Model: Recognizing Its Boundaries

Validation operates at two levels. Mathematical validation checks substitution, units, and calculation, while contextual validation examines the range of values, reliability of the data, and plausibility of the assumptions. In a fencing problem, 20 m of fencing is used to enclose three sides of a rectangle against a wall. If x denotes the side perpendicular to the wall, Equation (2) gives the area model and Figure 2 shows its graph.

$$S = x(20 - 2x) = -2x^2 + 20x \qquad 0 < x < 10 \tag{2}$$

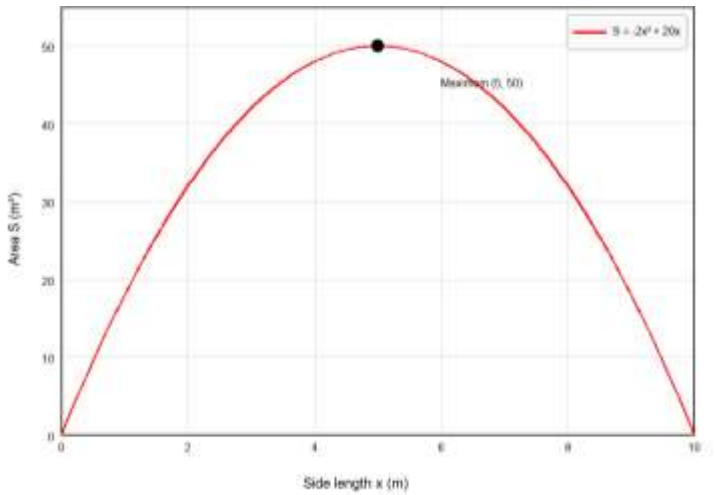


Figure 2. Area Function for a Three-Sided Enclosure against a Wall

A model remains incomplete if a student finds the maximum of the quadratic function without stating the domain of x. If the problem further specifies that no more than 8 m of wall is available or that 0.5 m of fencing is lost at the joins, the original expression must be adjusted. Such variations help students see that a model is not a copy of reality but an account of a relationship under a stated set of assumptions.

4.5 Transferring a Model: Carrying Structure into a New Problem

Transfer requires students to move beyond surface resemblance and attend to relational structure. Enclosure area, profit maximization, and the height of a projectile appear unrelated but may all lead to a quadratic function. Mobile-service plans, taxi fares, and tiered water prices may share a linear or piecewise-function structure. Rather than asking only which worked example a task resembles, teachers can ask students to name the common relationship—for instance, a constant rate of change or the product of two linearly constrained quantities—and then identify how parameters and constraints change in the new setting.

5. Reorganizing the Integrated Unit on Functions and Equations

5.1 Big Idea and Unit Objectives

The unit is organized around a big idea: quantitative relationships in real situations can be represented by equations and functions, and the form of a model depends on the investigative goal, relationships among variables, and relevant constraints. Content is therefore arranged not as a sequence of solution techniques but as a movement from a particular state to a changing process, then to model comparison and transfer. Equations are useful for finding an unknown state that meets specified conditions; functions describe relationships and patterns of change; graphs support the interpretation of solutions, intersections, extrema, and rates of change.

Six forms of performance express this big idea in the unit objectives: extracting variables and constraints from a situation; selecting an equation, function, graph, or table; moving among algebraic expressions, equations, and function graphs; interpreting parameters, solutions, intersections, and extrema in context; validating a model and revising it when conditions change; and transferring a familiar structure to a problem with a different surface context.

5.2 Five Types of Transfer Task

To make transfer an observable requirement rather than a verbal aspiration, the paper groups tasks into five types. Their design, contribution to transfer, and examples are shown in Table 2.

Table 2. Five Types of Transfer Task

Task type	Design	Contribution to transfer	Example
Same model, different context	Retains the relational structure while changing the real-world setting	Recognizing underlying structure	Taxi fares transferred to tiered water pricing
Same context, different models	Represents one context with equations, functions, and graphs/tables	Understanding the different purposes of models	Use an equation to find a break-even point and functions to compare usage ranges
Condition variation	Changes parameters, constraints, or data precision	Developing awareness of model revision	Adds a wall-length limit and material loss to a fencing task
Inference from data	Infers a relationship from a table or graph and then tests it	Developing attention to hypotheses and evidence	Uses motion data to compare linear and quadratic models
Model comparison	Offers two or more candidate models	Evaluating appropriateness and simplicity	Compares linear and piecewise predictions

5.3 Unit Learning Sequence

The unit can be organized in five phases. Familiar situations first reactivate students' experience with equations and focus attention on variables, equality relationships, and contextually valid solutions. Functions are then introduced to extend the study from one unknown value to an entire pattern of change. The third phase translates between equations and functions and interprets equation solutions through graph intersections. The fourth turns to quadratic relationships and optimization, with particular attention to domain and constraints. A final transfer project asks students to compare models, examine data, and prepare a written account. Interpretation and reflection are retained in every phase rather than postponed until the end.

6. Illustrative Transfer Tasks

6.1 Task 1: Choosing between Mobile-Service Plans and Finding the Break-Even Point

A mobile-service provider offers two plans. Plan A charges a monthly fee of 20 yuan plus 0.15 yuan per minute. Plan B charges a flat 50 yuan per month for up to 200 minutes and 0.10 yuan for each additional minute. Students first construct a function relating cost to call time for each plan, use an equation to find the break-even point, and then recommend a plan for different

ranges of monthly usage. The important issue is not the break-even value alone, but why Plan B requires a piecewise function, whether the models omit other charges, and how the graph supports the recommendation. Equation (3) gives the two cost models, and Figure 3 shows their piecewise graphs and point of intersection.

$$\begin{cases} A(x) = 20 + 0.15x & \\ B(x) = 50 & 0 \leqslant x \leqslant 200 \\ B(x) = 50 + 0.10(x - 200) & x > 200 \end{cases} \tag{3}$$

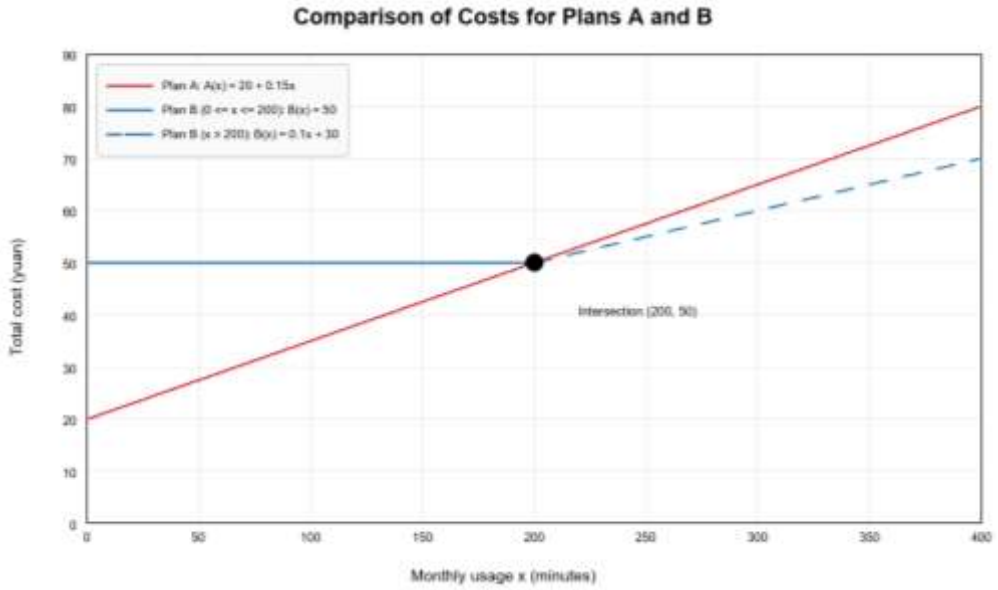


Figure 3. Cost Functions and Break-Even Point for Two Mobile-Service Plans

The question sequence can begin by distinguishing tariff rules from variables: Which quantities are fixed by the plans, and which change with usage? Why must Plan B be represented piecewise? What does it mean for the costs to be equal on the graph? If monthly call time fluctuates widely, is one break-even point sufficient for a decision? If data allowances are included, how should the comparison criteria change? This sequence connects equation solving, function graphs, and a practical decision.

6.2 Task 2: Competition among Fencing Models

In a fencing task, groups can investigate three alternatives: enclosing three sides against a wall, enclosing four sides away from a wall, and incorporating an existing 8 m section of fencing. Students state their assumptions, construct area functions, compare maximum areas and the conditions under which they are attained, and decide whether factors such as material loss or the width of an entrance should be included. The teacher does not prescribe a model in advance. Comparing the alternatives makes it clear that the same goal can lead to different representations when the conditions change. Equation (4) and Figure 4 show two representative models.

$$\begin{cases} S_1 = x(20 - 2x) \\ S_2 = x(10 - x) \end{cases} \tag{4}$$

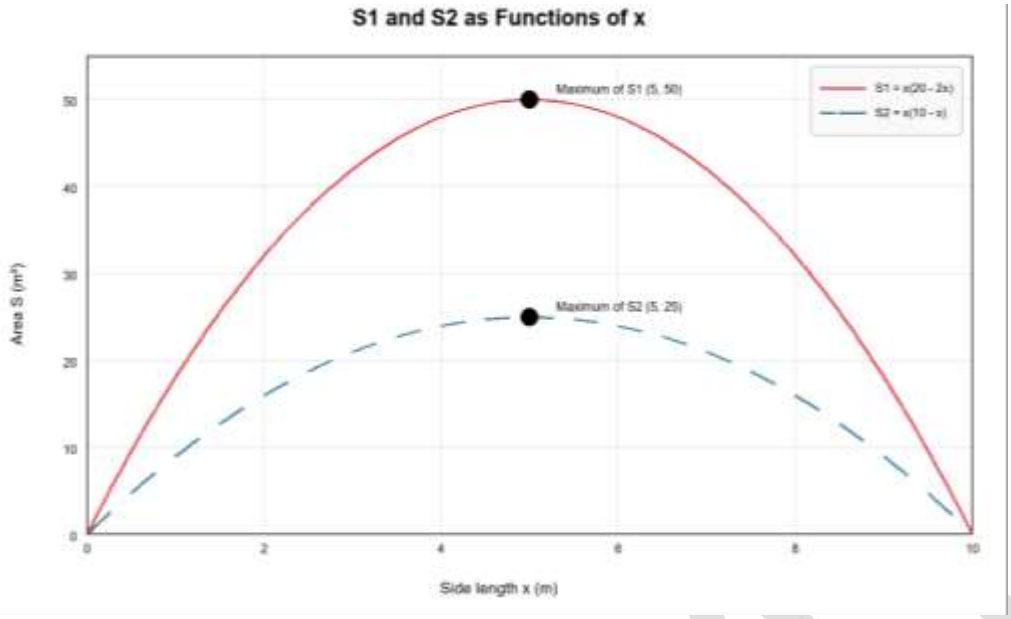


Figure 4. Comparison of Area Functions for Two Fencing Plans

For transfer, the context can be changed to pricing and sales volume: each one-yuan reduction in price is expected to increase sales by a certain number of units. Students identify the common structure—the product of two linearly changing quantities—while also explaining why the constraints from the fencing problem cannot simply be copied into a market setting. Sales, for example, may be limited by market capacity. The structure can be reused; the constraints must be reconsidered.

6.3 Task 3: Selecting a Model from Data

In this task, students are not told in advance which type of function to use. They receive distance records at several times after a vehicle starts, draw a scatterplot, and compare the increments over successive time intervals. If the increments are approximately constant, a linear model can be tried; if they change systematically, a quadratic or piecewise description may be more appropriate. Students then examine prediction errors and discuss how measurement error, driving conditions, and the sampled interval affect their judgment. The cycle of proposing a hypothesis, testing it against evidence, and revising it when necessary becomes visible. Equation (5) gives the mean absolute error.

$$e_i = y_i - \hat{y}_i \quad (5)$$

In Equation (5), e_i is the difference between the i-th observation and the corresponding model prediction. Junior secondary students do not need formal statistical inference here, but they can inspect the magnitude and direction of errors. Errors that grow consistently in one direction may indicate a systematic pattern omitted by the model. Small errors without an evident pattern show only that the model is serviceable over the current data range, not that it holds on every interval.

6.4 A Tiered Homework Sequence

Homework is organized at three levels according to the amount of prompting and openness. Foundational tasks ask students to identify variables, complete a model, and state units. Developmental tasks change one or two conditions and require a revision. Challenge tasks ask

students to collect household tariff or measurement data, pose a question, build a model, and discuss its limitations. The progression comes not from adding more computation, but from gradually removing prompts and transferring responsibility for choice and explanation to the student. Feedback on homework should not stop at marking the final answer. Students can keep a model passport that records the model's name, key relationship, conditions of use, common pitfalls, and possible transfer contexts. At the end of the unit, they compare two models and explain their similarities, differences, and cases in which each should not be used. The passport gathers experiences that would otherwise remain scattered and gives teachers evidence of whether students have developed a structural view.

7. Assessing the Mathematical Modelling Perspective

7.1 Principles of Assessment

Assessment should first align with the goals and tasks. If validation and transfer are part of the unit objectives, assessment cannot be limited to forming expressions and calculating. It should also retain evidence of the process and allow students to revise an initially immature representation into a more defensible model. Mathematical errors and modelling defects need to be distinguished: one computational slip does not erase a sound relational structure, while a correct calculation cannot substitute for judging the reasonableness of an assumption. In collaborative tasks, group products and individual understanding should both be examined so that the quality of the final product does not conceal differences among members.[18]

7.2 A Five-Dimension, Four-Level Rubric

On the basis of the progression above, Table 3 describes four levels of student performance across five dimensions. The levels are intended to represent developmental differences, not to be converted mechanically into a single aggregate score.

Table 3. Five-Dimension, Four-Level Rubric for the Mathematical Modelling Perspective

Dimension	Level 1: Prompt-dependent	Level 2: Basic performance	Level 3: Independent, reasoned performance	Level 4: Transfer and refinement
Context recognition	Has difficulty distinguishing variables from conditions	Extracts the main information	Identifies constraints and redundant information	Questions data and proposes necessary assumptions
Model representation	Can only imitate a given expression	Constructs one model	Moves among several representations	Compares models and selects the more appropriate one
Mathematical solution	Produces an incomplete solution	Completes routine calculations	Uses a correct method and communicates it clearly	Uses more than one method to verify a result

Dimension	Level 1: Prompt-dependent	Level 2: Basic performance	Level 3: Independent, reasoned performance	Level 4: Transfer and refinement
Interpretation and validation	Provides only a numerical answer	States a unit and a simple conclusion	Explains range, rejection of solutions, and assumptions	Analyses error and revises the model
Transfer and refinement	Cannot invoke the model when the context changes	Applies it to a closely similar task	Recognizes the structure and adjusts parameters	Rebuilds the model and explains its limits

7.3 Collecting Evidence and Providing Feedback

Evidence can be gathered across ordinary classroom activity. Worksheets preserve traces from an initial assumption to a revised model. Board work and oral reasoning help a teacher decide whether students understand the meaning of symbols. Modelling products show how they integrate information and design a solution; transfer tasks show whether they can independently invoke or adapt a model in a new context; reflective notes reveal their understanding of applicability and limitations. These materials answer different questions. Read together, they provide a more faithful account of students' modelling perspective than a single paper-and-pencil test. Feedback should identify a particular phase rather than simply label a student as weak at modelling. A teacher might write, for example, 'The cost function is correct, but the 200-minute breakpoint is not stated,' or 'Both roots were found, but the invalid one was not rejected using the length condition.' Anonymized classroom errors can also be compared: in one case the calculation is valid but the result is impossible in context; in another the data fit appears good, but the model is used outside its range. Such feedback makes the next revision more visible to the student.

8. Recommendations for Classroom Implementation

8.1 Use a Big Idea to Organize the Unit, Not a Collection of Exercise Types

An integrated unit is not created by adding content or placing several lessons side by side. Teachers need a big idea—here, the representation and selection of quantitative relationships—that runs through the unit, and they need to identify the contribution of each lesson to the five-stage progression. Practice with solution techniques remains necessary, but it should be embedded in the use of models so that students understand the distinct roles played by formulas, graphs, and calculations in the modelling process.

8.2 Make Assumptions, Interpretation, and Revision Routine

Several questions can become regular classroom prompts: What has this model left out? What does the symbol mean in the situation? Are the unit and range of the result reasonable? Which part must change when a condition changes? Is there a simpler or more appropriate representation? With repeated use, these external prompts can gradually become internal habits of thought.

8.3 Control Contextual Complexity and Gradually Withdraw Scaffolds

A realistic context is not necessarily one with more information. At the beginning, teachers can use quasi-realistic problems with clear relationships and provide a variable table, diagram, or question sequence. Redundant information, uncertain data, and multi-objective choices can be introduced later as some scaffolds are removed. The principal difficulty should come from selecting and interpreting a model, not from background knowledge wholly unfamiliar to students.[19]

8.4 Use Visualization and Digital Tools Judiciously

Graphing software, spreadsheets, and dynamic geometry tools can display parameter changes, intersections, and errors quickly and reduce repetitive drawing. Students should, however, make an initial judgment before turning to a tool: they first propose a model or prediction, then use the tool to examine it and explain what appears on the screen. If automatic fitting is used without discussing the meaning of variables or the range of applicability, visualization can hide rather than reveal the modelling process.

8.5 Build a Classroom Culture in Which Models May Fail

A failed model is not merely an error; it can be productive instructional material. A linear model that works over a short interval but distorts a longer one makes the importance of range visible. Two groups may also produce different models because they adopted different assumptions, not because one must be wrong. Teachers can place such models side by side and ask students to retain, modify, or abandon them on the basis of evidence rather than rush toward a single standard form.

9. Conclusion

This paper places model transfer at the upper end of the proposed progression in order to address the distance between being able to construct and solve with a model and being able to use it in a new situation. When students face an unfamiliar problem, their ability to identify quantitative relationships, adjust parameters, and reconsider constraints provides stronger evidence of structural understanding than their ability to reproduce the steps of a worked example.

The five-dimension, four-level rubric brings contextual recognition, model representation, mathematical solution, interpretation and validation, and transfer and refinement into one framework. Its purpose is not to attach a deceptively precise score to a core competency, but to translate an abstract goal into classroom performances that can be observed and recorded, allowing teachers to locate the phase in which a student needs support. The model passport is a process tool designed to accompany the rubric. By recording key relationships, conditions of use, common pitfalls, and possible transfer contexts, students can reconnect experiences distributed across separate chapters. Teachers can use the same record to trace change and decide what kind of task or scaffold should follow. The value of an integrated-unit perspective lies not in enlarging the scale of instruction, but in allowing the same structural ideas to reappear, be compared, and be revised. Identification, construction, interpretation, validation, and transfer can then become one continuous developmental thread. The framework, tasks, and rubric proposed here remain design proposals rather than empirical evidence of effectiveness. They require further examination and adjustment through students' work, classroom dialogue, and transfer performances in actual lessons.

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